

Table 2.1 Properties of some special distributions

Handout #3

	Survivor function	Density function	Hazard	No. of parameters
✓ (i) Exponential	$e^{-\rho t}$	$\frac{\rho e^{-\rho t}}{\Gamma(k)}$	$\rho$	1
(ii) Gamma	incomplete gamma function	$\frac{\rho(\rho t)^{k-1} e^{-\rho t}}{\Gamma(k)}$	—	2
✓ (iii) Weibull	$\exp[-(\rho t)^k]$	$\kappa \rho (\rho t)^{k-1} \exp[-(\rho t)^k]$	$\kappa \rho (\rho t)^{k-1}$	2
(iv) Gompertz-Makeham	—	—	$\rho_0 + \rho_1 e^{\rho_2 t}$	3
(v) Compound exponential	$\frac{(\kappa/\rho_0)^k}{(t + \kappa/\rho_0)^k}$	$\frac{\kappa(\kappa/\rho_0)^k}{(t + \kappa/\rho_0)^{k+1}}$	$\frac{\kappa}{t + \kappa/\rho_0}$	2
(vi) Orthogonal polynomial	$e^{-\rho t} [1 + \kappa_1 \rho t + \kappa_2 \rho t(\rho t - 2)]$	$\rho e^{-\rho t} [1 + \kappa_1 L_1(\rho t) + \kappa_2 L_2(\rho t)]$	—	2
(vii) Log normal	—	—	nonmonotonic	2
(viii) Log logistic	$[1 + (t\rho)^k]^{-1}$	$\kappa \rho^k t^{k-1} [1 + (t\rho)^k]^{-2}$	$\frac{\kappa t^{k-1} \rho^k}{[1 + (t\rho)^k]}$	2
(ix) Generalized F	—	—	—	2
(x) Inverse Gaussian	—	—	—	4
(xi) Translation	—	—	—	2
(xii) Scale family	$\mathcal{G}(\rho t)$	$\rho g(\rho t)$	$\rho h^{(g)}(\rho t)$	1 extra for origin 1 extra for scale
(xiii) Proportional hazard family	$[\mathcal{L}(t)]^\psi$	$\psi [\mathcal{L}(t)]^{\psi-1} l(t)$	$\psi h^{(\psi)}(t)$	1 extra for proportionality

Table 2.2 Some properties useful in assessing distributional form

	$\log h(t)$	$H(t)$	$\log H(t)$	Coefficient of variation
Is it constant? exponential	—	linear in $t$ ? exponential	—	1? exponential
Is it linear in $t$ ? Gompertz ( $\rho_0 = 0$ )	—	—	linear in $t$ ? Gompertz ( $\rho_0 = 0$ )	< 1? Gamma ( $\kappa > 1$ ), Weibull ( $\kappa > 1$ ) Log normal ( $\tau < 0.83$ ), Log logistic ( $\tau < 0.118$ )
Is it linear in $\log t$ ? Weibull	—	—	linear in $\log t$ ? Weibull	—
Is it nonmonotonic? Log normal Log logistic	—	—	asymptotically linear in $t$ ? Distribution with exponential tail	> 1? Gamma ( $\kappa < 1$ ), Weibull ( $\kappa < 1$ ) Log normal ( $\tau > 0.83$ ), Log logistic ( $\tau > 0.118$ ) Compound exponential