BIOSTAT/STAT 576
Statistical Methods for Survival Data

Problem Set 3

1. Suppose that $\hat{S}(t) = \prod_{s \le t}\{1 - \Delta N(s)/Y(s)\}$ is the Kaplan-Meier estimator. Show that

$$\hat{S}(t) - S(t) = -S(t) \int_0^t \frac{\hat{S}(u-)}{S(u)} \frac{dM(u)}{Y(u)},$$

where $M(t) = N(t) - \int_0^t Y(t) d\Lambda(t)$.

2. Assume that $\Pr\{T < C\} > 0$ and $\Pr\{T \le \tau\} = 1$ for some finite τ. Let the nonparametric estimator of $m(t) = E(T - t \mid T > t)$ be

$$\hat{m}(t) = \frac{1}{\hat{S}(t)} \int_t^\tau \hat{S}(u) du.$$

Establish the consistency and weak convergence of $\hat{m}(\cdot)$, and describe a simulation-based method to construct 95% pointwise confidence intervals and confidence bands for $m(\cdot)$.

3. For the PBC data, estimate $S(t)$ by $\exp[-\hat{\Lambda}(t)]$ for each treatment group, where $\hat{\Lambda}(t)$ is the Nelson-Aalen estimator. Then apply the direct, bootstrap and simulation methods to calculate 95% pointwise confidence intervals and confidence bands for $S(t)$. Plot $\hat{S}(t)$ and its associated confidence intervals and confidence bands. Note: Bootstrap and simulation are implemented 1000 times in calculation.