BIOSTAT/STAT 576
Statistical Methods for Survival Data

Problem Set 5

1. Let the standardized weighted Log-rank statistic be

$$W = \frac{\sum_{i=1}^{n} \int_0^{\tau} W(u)\{Z_i - \bar{Z}(u)\} dN_i(u)}{\sum_{i=1}^{n} \int_0^{\tau} W(u)^2 \bar{Z}(u)\{1 - \bar{Z}(u)\} dN_i(u)^{1/2}},$$

where Z_i is binary of 0 or 1, $\bar{Z}(u) = \sum_{i=1}^{n} Z_i Y_i(u) / \sum_{i=1}^{n} Y_i(u)$ and $W(u)$ is some weight function that converges to $w(u)$. Consider the local alternatives H_{1n} to be specified by the so-called additive mean residual life model, i.e.,

$$m(t \mid Z) = m_0(t) + \beta_n Z,$$

where $m(\cdot)$ and $m_0(\cdot)$ are the mean residual life functions and $\{\beta_n\}$ are the parameters such that $n^{1/2}\beta_n \rightarrow \xi$.

   (a) Derive a class of weight functions for $W(u)$, $W_A(u)$ say, so that W would be most powerful against the specified H_{1n}.

   (b) Derive a sample size formula based on $W_A(u)$ against a fixed alternative of $m(t \mid Z) = m_0(t) + \beta_0 Z$.

2. One of the advantages for the proportional hazards model is its extension in dealing with time-dependent covariates. Suppose that $Z_i(t)$ is a time-dependent covariate for the ith subject, $i = 1, 2, \ldots, n$. An extended proportional hazards model, termed as relative risk (Cox) regression model (Kalbfleisch & Prentice, p. 96), would usually assume that

$$\lambda(t \mid Z(t)) = \lambda_0(t) \exp\{\beta Z(t)\},$$

where β is the parameter. Use the Quasi Partial Scoring (QPS) to derive an estimating function for the regression parameter β.

3. In general, the term 'relative risk' is reserved for comparing two cumulative distributions functions of time-to-event, or equivalently, two cumulative hazard functions. Instead of the extended Cox regression model, let's consider the following relative risk model specified in cumulative hazard functions

$$\Lambda(t \mid Z(t)) = \Lambda_0(t) \exp\{\beta Z(t)\}.$$

Suppose that β is known. Use the QPS approach to derive an estimating function for $\Lambda_0(t)$.

(Note: In fact, this QPS estimating function has a closed-form solution for $\Lambda_0(t)$. If you work out the solution, with some extra effort, this should lead to a decent piece of methods/theory work with wide applications.)